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STATIC CHARACTERISTICS OF EQUILIBRIUM TWO-PHASE TRANSPIRATION COOLING SYSTEMS

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Abstract—The problem is formulated on steady two-phase cooling of a homogeneous porous plate. A close relationship is shown to exist between thermal and hydrodynamic components of the process governed by a non-linear closed system of differential equations. The laws of filtration movement of a coolant with its phase conversions inside a porous wall are analysed.

On the basis of the results obtained from a solution of the system of equations describing two-phase transpiration cooling static characteristics of the process are plotted. These are pressure vs temperature curve in the suggested region of phase conversions, hydrodynamic and thermal characteristics. investigation of a periodic stability of the process with the use of a set of these characteristics makes it possible to find permissible disturbances of all of the characteristic parameters and reveal peculiarities of steady two-phase transpiration cooling.

NOMENCLATURE

X, x,	dimensional and non-dimensional coordinates;
G, g,	dimensional and non-dimensional mass flows rates of coolant;
L, l,	dimensional and non-dimensional coordinates of phase conversion region;
δ,	wall thickness;
P_0,	delivery pressure;
P_1,	ambient pressure;
q,	density of external heat flux;
T_0, T_e, T_1,	temperatures at characteristic points;
$\theta_0, \theta_1, \theta_2$,	non-dimensional temperatures;
α, β,	viscous and inertial resistance coefficients;
Π,	porosity of wall material;
λ,	thermal conductivity;
μ,	dynamic viscosity;
ν,	kinematic viscosity;
ρ,	density;
c,	heat capacity;
r,	total heat of vaporization;
i,	enthalpy;
Re,	Reynolds number of coolant flow.

Subscripts

0, 1, 2,	parameters for coolant flow regions, namely: liquid, porous material–liquid, porous material–vapour, respectively;
e,	parameters for suggested region of phase conversions;
s,	parameters in a saturated state.

Superscripts

$'$, $''$,	liquid and vapour properties in a saturated state, respectively.

INTRODUCTION

IMPROVEMENT of working parameters in such devices as combustion chambers and nozzles of rockets, combustion chambers and blades of gas-turbine engines, MHD generators, and front sections of reentry vehicles sets forth increasing requirements to the reliable systems of heat protection. Cooling with liquid under phase conversions inside a porous heated plate appears to be most promising since it possesses a number of important advantages such as minor specific volume and high heat absorbing capacity of a coolant due to liquid evaporation and vapour superheat; absence of limitations for aerodynamic shear stresses over the external surface and the density of supplied heat flux that may be convective, radiative or convective–radiative; an increase of external surface temperature within reliable limits contributes both to reduction of the heat flux supplied and to thermal radiation of some amount of heat into the ambient medium.

The above advantages of the heat protection method are not fully employed since it possesses a great drawback as well: two-phase transpiration cooling can be unsteady [1–11]. In an unstable system of two-phase transpiration cooling it is impossible to stabilize the evaporation region inside a porous structure: a slight increase in heat flux or reduction of wall pressure drop results in quick displacement of the evaporation region from an external wall surface towards the internal one. The replacement of a liquid flow by a one-phase vapour flow decreases the flow rate that involves the wall burnout at high heat fluxes.

Instability in operation of transpiration cooling installations makes it impossible to reveal the laws of a steady-state process for heat transfer and resistance of a filtering coolant under phase conversions. A study of steady two-phase transpiration cooling becomes

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*Deceased.

more difficult since it involves such little-known problems as determination of thermophysical and hydraulic properties of porous ceramic materials; heat transfer and resistance at filtration of on e-phase heat-transfer agents, heat transfer with an internal permeable surface of the cooled wall in a longitudinal flow. The lack of data concerning the main laws of steady two-phase transpiration cooling [6–8], [11–12] compels us to use the model that simplifies a real process but does not change its quality.

STATEMENT AND SOLUTION OF THE PROBLEM ON EQUILIBRIUM TWO-PHASE TRANSPIRATION COOLING

The proposed equilibrium model of the process is shown in Fig. 1. A homogeneous flat porous plate, whose length and width are large as compared to thickness δ, is subjected to the action of a constant external heat flux with density q. Liquid with initial temperature T_∞ is used as a coolant. Under the action of delivery pressure P_0 the coolant is forced through the wall into the ambient medium filled with coolant vapour at prescribed pressure P_1. As liquid flows inside the porous structure it absorbs heat and its temperature grows. In this case the local equality of coolant and material temperature, i.e. thermal equilibrium survives. Simultaneously with temperature growth pressure drop in liquid is observed. As soon as liquid pressure and temperature achieve their magnitudes corresponding to the saturation state, its phase conversion proceeds. Evaporation occurs in a thin, as compared to the plate thickness, zone at L distance from the internal surface. Pressure difference over the interface due to surface tension effects is assumed to be small in comparison with the pressure drop on the plate. The generated vapour absorbs heat between the evaporation zone and external surface and when superheated flows into the ambient medium.

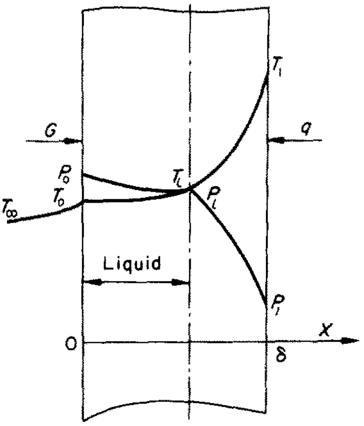

FIG. 1. Physical model of two-phase transpiration cooling.

With account taken of the assumptions made a steady-state two-phase transpiration cooling of a homogeneous plate is described by a system of differential equations that comprises: the continuity equations of a coolant flow

$$\frac{\mathrm{d}G}{\mathrm{d}X} = 0; \tag{1}$$

the liquid motion equation in porous media, i.e. the modified Darcy equation

$$-\frac{\mathrm{d}P}{\mathrm{d}X} = \frac{\alpha\mu}{\rho} G + \frac{\beta}{\rho} G^2, \tag{2}$$

and the equation of energy

$$\lambda' \frac{\mathrm{d}^2 t_0}{\mathrm{d}X^2} - G.c' \frac{\mathrm{d}t_1}{\mathrm{d}X} = 0 \tag{3}$$

for liquid at $0 > X > -\infty$

$$\lambda_1 \frac{\mathrm{d}^2 t}{\mathrm{d}X^2} - G.c' \frac{\mathrm{d}t_1}{\mathrm{d}X} = 0 \tag{4}$$

for the liquid–porous body region $0 < X < L$

$$\lambda_2 \frac{\mathrm{d}^2 t_2}{\mathrm{d}X^2} - G.c' \frac{\mathrm{d}t_2}{\mathrm{d}X} = 0 \tag{5}$$

for the vapour–porous structure region $L < X < \delta$.

The boundary conditions reveal the following peculiarities of the process: an initial liquid temperature is prescribed as

$$X = -\infty \quad t_0 = T_\infty. \tag{6}$$

Conjugation conditions of solutions are fulfilled at the region boundaries; here at the interface a heat flux is decreased by the magnitude necessary for complete evaporation of the whole coolant flow

$$X = 0, \quad t_0 = t_1 = T_0, \quad \lambda' \frac{\mathrm{d}t_0}{\mathrm{d}X} = \lambda_1 \frac{\mathrm{d}t_1}{\mathrm{d}X} \tag{7}$$

$$X = L, \quad t_1 = t_2 = T_e, \quad \lambda_2 \frac{\mathrm{d}t_2}{\mathrm{d}X} - \lambda_1 \frac{\mathrm{d}t_1}{\mathrm{d}X} = Gr \tag{8}$$

At the external wall surface the heat flux density, ambient pressure and maximum temperature of reliable operation of porous material are found

$$x = \delta, \quad q = \lambda_2 \frac{\mathrm{d}t_2}{\mathrm{d}X}; \quad P = P_1, \quad T_1 \leqslant T_1^{**}. \tag{9}$$

The condition of thermodynamic equilibrium of phase conversion affects the position of the interface and makes the solution of the problem completed

$$X = L, \quad P_e = P_s(T_e). \tag{10}$$

When writing down the boundary conditions, the unknown temperatures of the region boundaries are designated as T_0, T_e, T_1. The independent boundary conditions of the problem are: maximum permissible temperature of external surface T_1^{**} for the equation of continuity; ambient pressure for the equation of motion; initial temperature of liquid and density of an external heat flux for the second-order equations of energy.

The modified Darcy equation regards both for friction resistance (the first term in the r.h.s. of the equation) and inertial resistance (the second term), when liquid flows in porous material. Resistance coefficients (viscous coefficient α and inertial coefficient β) characterize porous structure and depend neither on the kind of filtering liquid [13–14] nor on the material temperature [14–17]. Ceramet materials produced by pressing followed by sintering of metal powder appear to be the most promising when designing cooling systems. The resistance coefficients of such a material depend on its porosity, dimensions and shape of particles of the original powder. For the moment some correlations are obtained between porosity and resistance coefficients for different ceramic materials. The elaborated processing methods allow porous structures with the required hydraulic properties to be obtained [17–22].

In a general case effective thermal conductivity of disperse two-phase systems depend both on the structure and kind of porous material and on the properties of a liquid filler. There are but scanty data on thermal conductivity of two-phase media consisting of porous liquid-filled ceramet materials available only for systems with a gas-filler [23–28]. From these experimental data it follows that thermal conductivity of such materials is a characteristic being satisfactorily calculated with the help of the relationship obtained by Odelevsky [29] and Skorokhod [30].

An analysis of expressions (1–10) shows that even a two-phase transpiration cooling model is described by a non-linear closed system of differential equations. Really, physical properties of a coolant, entering into the equations of energy and motion, depend on temperature and pressure and spasmodically change within a region with phase conversions. The position of this region in its turn is defined from the solution of the above equations provided that the nonlinear boundary conditions of equilibrium phase conversion (10) and complete evaporation of a coolant (8) with unknown mass flow rate are fulfilled.

The following method is proposed for the solution of this system: assuming the position of the interface to be known, pressure and temperature in the supposed region with phase conversions are found from the equations of motion and energy. The values obtained are presented as points on the phase diagram of the coolant. A set of such points corresponding to all the positions of the phase conversion surface inside a porous wall form a continuous curve. From mutual behaviour of the curve obtained and that of saturation, a conclusion is drawn up concerning the position of the evaporation region at the stability of the system considered, the latter being more important.

The results are expressed in an analytical form if physical properties of each phase of the coolant and wall material are assumed to be constant. This, however, does not violate the conclusion on the stability of the process. When choosing physical properties of liquid and vapour in a system with phase conversions, the saturation state at a pressure equal to the known ambient pressure P_1 seems to be mostly appropriate feature. In this case system (1–10) permits separation of the thermal and hydrodynamic components of the process.

The specific of a two-phase transpiration cooling process is attributed to the hydrodynamics of a coolant with phase conversions inside a porous wall. Since the problem has not been thoroughly investigated up to nowadays some of its most important properties should be pointed out. Integration of the equations of continuity and motion with respect to both phases of the coolant flow yields, after reduction to the dimensionless form, an equation of motion

$$1 = gm + g^2 nRe, \tag{11}$$

a solution of the equation of motion

$$g = \frac{-1 + \sqrt{\left(1 + 4Re\dfrac{n}{m^2}\right)}}{2Re\dfrac{n}{m}} \tag{12}$$

relative pressure drop within the vapour region

$$\frac{P - P_1}{P_0 - P_1} = 1 - gl - g^2 lRe. \tag{13}$$

In the relationships above a use is made of the dimensionless coordinate of the phase conversion $l = L/\delta$ and additional complexes

$$m = \left[l + \frac{v''}{v'}(1 - l)\right], \quad n = \left[l + \frac{\rho'}{\rho''}(1 - l)\right] \tag{14}$$

non-dimensional coolant flow rate

$$g = \frac{G}{G_l} \tag{15}$$

which is the ratio of the actual flow rate G to that of a liquid coolant for the Darcy flow

$$G_1 = \frac{P_0 - P_1}{\delta v' \alpha}. \tag{16}$$

Reynolds number of the flow as a measure of the ratio of inertial resistance forces to viscous ones when a coolant flows in a porous medium

$$Re = \frac{G_1 \beta/\alpha}{\mu'} = \frac{(P_0 - P_1)\beta/\alpha}{\delta v' \alpha \mu'}. \tag{17}$$

When composing the Reynolds number, the characteristic dimension of the porous structure β/α being the ratio of appropriate resistance coefficients is used. The mode of Reynolds number calculation assumed here allows determination of the most important characteristic of the liquid flow with the aid of pressure drop over the plate.

The actual coolant flow rate and required pressure drop over the plate are found from the expressions

$$G = \frac{\mu'}{\beta/\alpha}\frac{m}{2n}\left[-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right] \tag{18}$$

$$P - P_1 = \delta\left\{G\alpha[v'l + v''(1 - l)] + G^2\beta\left[\frac{l}{\rho'} + \frac{(1 - l)}{\rho''}\right]\right\}. \tag{19}$$

Hence it follows that the flow rate of the coolant chosen depends on the position of the phase conversion surface, physical properties of its both phases controlled by the ambient pressure and on the Reynolds number of the flow.

To have a clear idea of the results obtained, we shall consider, as an example, water filtration with phase conversion inside a porous stainless steel plate with thickness $\delta = 5$ mm and porosity $\Pi = 0{\cdot}2$. The resistance coefficients of the plate ($\alpha = 3{\cdot}5 \times 10^{13}\,\mathrm{m}^{-2}$, $\beta = 1{\cdot}2 \times 10^{8}\,\mathrm{m}^{-1}$) are chosen in accordance with the data of [17].

Figure 2 shows an effect of ambient pressure on the curve of dimensionless flow rate vs the position of the phase conversion surface for constant pressure drop over the plate (constant Reynolds number). It is manifest that a sharp decrease in the flow rate with recession of the region of evaporation from the external surface of the plate is peculiar to the process at small pressure, in particular. This is one of the main reasons of the instability of the process.

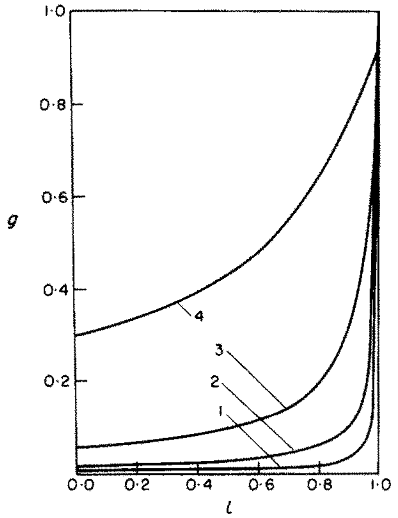

FIG. 2. Dimensionless coolant flow rate vs location of phase conversion region at $Re = 0{\cdot}1$; 1, $P_1 = 0{\cdot}1$ bar; 2, $P_1 = 1{\cdot}0$ bar; 3, $P_1 = 10$ bar; 4, $P_1 = 100$ bar.

According to the shape of the dimensionless liquid flow rate vs the Reynolds number curve (Fig. 3) the flow in porous material may be divided as: viscous or Darcy flow $Re < 0{\cdot}01$, transient flow $0{\cdot}01 < Re < 100$ and the inertia flow $Re > 100$.

Besides the flow rate reduction, an increase in the inertial resistance involves a change in the shape of the curve of dimensionless coolant flow rate vs the position of the phase conversion surface (that follows from Fig. 4) and further growth of a vapour contribution to the total resistance of the plate (Fig. 5).

The results presented in Fig. 5 acquire a particular importance as far as the reverse effect of vapour and liquid flows on the process stability is concerned. Thus, the viscosity of liquid falls with the temperature growth while it increases for vapour. When one-phase liquid is used for cooling a small increase in heat flux raises the wall temperature and, therefore, lowers the liquid viscosity. Flow drag decreases, and the increasing

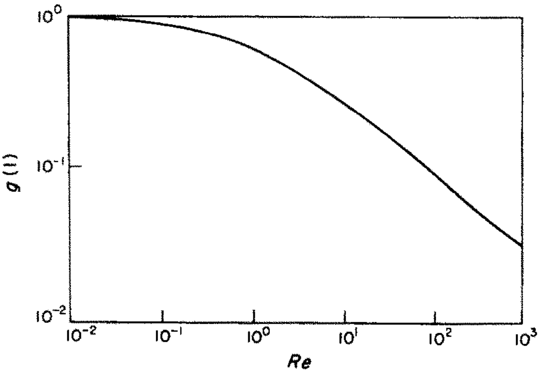

FIG. 3. Dimensionless liquid coolant flow rate vs Reynolds number.

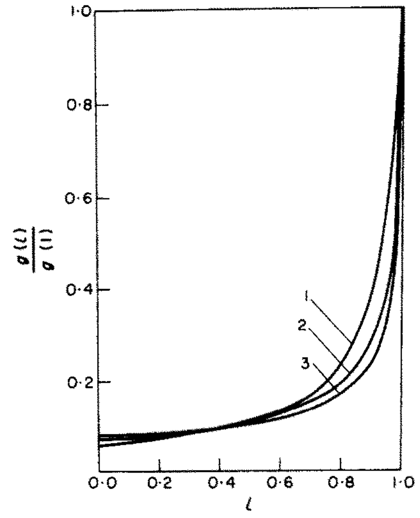

FIG. 4. Relative coolant flow rate vs position of phase conversion region at $P_1 = 10$ bar: 1, $Re = 0{\cdot}0$; 2, $Re = 1{\cdot}0$; 3, $Re = \infty$.

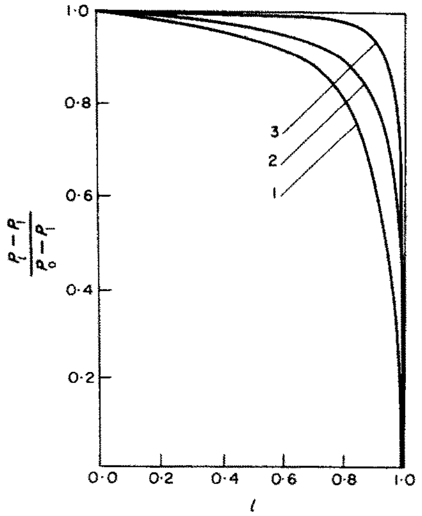

FIG. 5. Relative pressure drop within vapour region vs position of phase conversion region at $P_1 = 10$ bar: 1, $Re = 0{\cdot}0$; 2, $Re = 1{\cdot}0$; 3, $Re = \infty$.

liquid flow rate brings the system back to the state of equilibrium. The phenomenon is reverse when vapour flows through a wall. A slight increase in the temperature causes reduction of the vapour flow rate that contributes to further growth of the wall and vapour temperature.

For the fixed supposed position of the phase transition surface and constant physical properties of each phase of the coolant and solid material, the thermal aspect of the problem becomes linear. Distribution of dimensionless temperatures within three regions of a coolant flow is of the uniform form

$$\theta_0 = \frac{t_0 - T_\infty}{T_0 - T_\infty} = \exp\left(\frac{G\delta c'}{\lambda'} x\right);$$

$$\theta_1 = \frac{t_1 - T_0}{T_e - T_0} = \frac{\exp\left(\frac{G\delta c'}{\lambda_1} x\right) - 1}{\exp\left(\frac{G\delta c'}{\lambda_1} l\right) - 1}; \tag{20}$$

$$\theta_2 = \frac{t_2 - T_e}{T_1 - T_e} = \frac{\exp\left[\frac{G\delta c''}{\lambda_2}(x - l)\right] - 1}{\exp\left[\frac{G\delta c''}{\lambda_2}(1 - l)\right] - 1};$$

wherein temperatures at the boundaries of the boundaries T_0, T_e, T_1 and dimensionless coordinate $x = X/L$ are used. Of greatest importance in further analysis is the temperature of the supposed region of phase conversions and that of an external plate surface. To predict them, the following relationships are obtained

$$T_e = T_\infty + \frac{q \exp\left[\frac{G\delta c''}{\lambda_2}(l - 1)\right] - Gr}{Gc'} \tag{21}$$

$$T_1 = T_e + \frac{q\left\{1 - \exp\left[\frac{G\delta c''}{\lambda_2}(l - 1)\right]\right\}}{Gc''} \tag{22}$$

ANALYSIS OF TWO-PHASE TRANSPIRATION COOLING STABILITY USING TEMPERATURE DEPENDENCE OF PRESSURE IN THE SUPPOSED PHASE CONVERSION REGION

As a result of an individual solution of thermal and hydrodynamic components of the two-phase transpiration cooling process the expressions are obtained for calculation of pressure (13) and temperature (21) in the supposed region of phase conversions. It follows from these relations that the parameters within the region of phase conversions depend not only on constant characteristics of the system but also on the position of the region itself since its coordinate [1] enters into these expressions directly and through the value of the flow rate $G(l)$. The supposed coordinate of the phase transition surface is a parameter and may assume all the values from the interval $l \in (0, 1)$. The established parametric relationship in a brief form reads as

$$P_e = P_e(T_e). \tag{23}$$

A concrete form of this relationship depends on the parameters of the process. Examine some variants for small Reynolds numbers of a coolant flow. The following parameters are assumed to be constant: water-coolant with initial temperature $T_\infty = 20$°C; ambient pressure $P_1 = 10$ bar; wall thickness, $\delta = 5$ mm, porosity $\Pi = 0{\cdot}2$. The coefficients of viscous and inertial resistance $\alpha = 3{\cdot}5 \times 10^{13}\,\text{m}^{-2}$ and $\beta = 1{\cdot}2 \times 10^{8}\,\text{m}^{-1}$, respectively, and characteristic dimension of the porous structure $\beta/\alpha = 3{\cdot}44 \times 10^{-6}$ are calculated on the basis of experimental data for a plate of similar thickness and porosity, made of powder-sintered stainless steel [17]. The plate is cooled in such a way that at constant Reynolds numbers $Re = 0{\cdot}1$ phase transition takes place for the region with the coordinate $l = 0{\cdot}95$. Here the specific coolant flow rate $G = 1{\cdot}9\,\text{kg/m}^2\,.\,\text{s}$, the required total pressure drop over the plate $P_0 - P_1 = 1{\cdot}25$ bar and the pressure drop over the vapour region $Pe - P_1 = 0{\cdot}567(P_0 - P_1)$. The saturation temperature $T_s = 182{\cdot}8$°C corresponds to the pressure $Pe = 10{\cdot}7$ bar of the region with phase conversions.

Consider three cases with different density of an external heat flux: $q_{\text{I}} = 3{\cdot}5 \times 10^7\,\text{W/m}^2$, $q_{\text{II}} = 1{\cdot}8 \times 10^7\,\text{W/m}^2$, $q_{\text{III}} = 8{\cdot}3 \times 10^6\,\text{W/m}^2$. The effective thermal conductivity of the porous body–vapour medium, at which a phase transition in the region with the coordinate $l = 0{\cdot}95$ is equilibrium, are calculated from relation (21) at $T_e = T_s = 182{\cdot}8$°C and their values are: $\lambda_{2\text{I}} = 0{\cdot}65$ W/m . deg; $\lambda_{2\text{II}} = 1{\cdot}0$ W/m . deg; $\lambda_{2\text{III}} = 2{\cdot}6$ W/m . deg.

For the cases considered curves 1–3 in Fig. 6 plot the pressure vs temperature in the supposed region of phase conversions. The pressure within the phase conversion region calculated in the dimensionless form according to equation (13) is taken as one of the coordinates. For illustration appropriate dimensionless coordinates of the phase conversion surface are given. At constant Reynolds numbers the relations between the dimensionless pressure and coordinate are the same for the cases considered. Dimensionless temperature is the second coordinate. This form of temperature

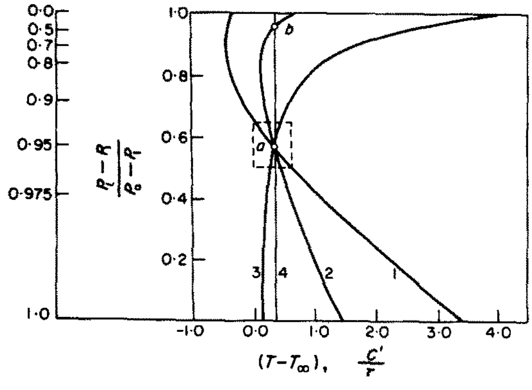

FIG. 6. Pressure vs temperature in supposed region of phase transition: 1, $q = 3{\cdot}5 \times 10^7\,\text{W/m}^2$; $\lambda_2 = 0{\cdot}65$ W/m . deg; 2, $q = 1{\cdot}8 \times 10^7\,\text{W/m}^2$, $\lambda_2 = 1{\cdot}0$ W/m . deg; 3, $q = 8{\cdot}3 \times 10^6\,\text{W/m}^2$, $\lambda_2 = 2{\cdot}6$ W/m . deg; 4, saturation curve.

expression is convenient for the supposed region with phase conversions since expression (21) is reduced to the form

$$(T_e - T_\infty)\frac{c'}{r} = \frac{q\exp\left[\dfrac{G\delta c''}{\lambda_2}(l-1)\right]}{Gr} - 1. \tag{24}$$

In the coordinates considered saturation curve 4 is practically a vertical line since an increase in the saturation temperature $T_s = 5{\cdot}2^\circ\mathrm{C}$ corresponds to the pressure drop $P_0 - P_1 = 1{\cdot}25\,\mathrm{bar}$ over the plate. In dimensionless coordinates $\Delta T_s(c'/r) = 0{\cdot}011$.

All three curves are intersected with the saturation line at the working point *a*, however, their slopes at this point and further behaviour are different. Also, curve 2 is intersected with the saturation line at the point *b*. In this case a solution is ambiguous.

For an unambiguous choice of the position of a phase conversion surface a relationship is to be found between the system stability and kind of intersection of the saturation line with the pressure vs temperature curve in the supposed region of phase conversions. Let in systems 1 and 3 external heat flux be suddenly decreased by a small value (by 5 per cent) against its original value. In the supposed region of phase conversions the temperatures pertinent to the new developed state decrease and assume the values shown by dash lines 1 and 3 in Fig. 7. At the moment of changing external heat flux the liquid–vapour interface is in a state shown by the point *a*. With reduced heat flux the amount of heat supplied to the phase transition region is insufficient for complete evaporation of the whole coolant flow. If liquid evaporated completely, the temperature within the evaporation region shown by the point *a*, would be lower than that of saturation (point *a*). This causes the phase transition region to propagate towards the external plate surface that involves an increase in the coolant flow rate (curve 3 in Fig. 2 illustrates the corresponding change in the flow rate). Further behaviour of the process may be inferred from the kind of intersection of saturation line 4 with curves 1 and 3 at the point *a*.

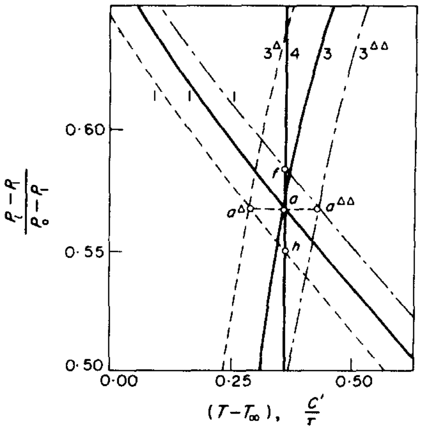

Fig. 7. Scaled up element of Fig. 6: solid lines 1 and 3 refer to undisturbed state; dashed lines refer to steady state with slightly decreased heat flux; dot-and-dash lines refer to steady state with increased heat flux.

From equation (22) written in the form

$$q\exp\left[\frac{G\delta c''}{\lambda_2}(l-1)\right] = q - Gc''(T_1 - T_e) \tag{25}$$

it follows that the numerator in the first summand in the r.h.s. of expression (24) for temperature in the supposed region of phase transition is recognized as the density of heat flux supplied to this region. If with displacement of the phase conversion surface the amount of heat supplied to it increases at a higher rate than that necessary for evaporation of a sharply increasing coolant flow [denominator in the r.h.s. of (24)], then the temperature in the supposed region with phase transition rises and a new steady state sets up with equilibrium phase transition. In system 1 the interface slightly displaces and takes a new position shown by the point *h*. Conventionally the transition into a new state is shown by the line $a - a^\Delta - h$. In system 3 the amount of heat supplied to the moving interface increases slower than that necessary for evaporation of the rapidly growing coolant flow. The phase transition region moves until it reaches the external surface of the plate. In this case the coolant flow rate exceeds the value determined from the condition of complete evaporation.

In case of slightly increasing heat flux density the interface in system 1 will recess and take a new position shown by the point *f*. In system 3 a slight increase in the heat flux results in continuous propagation of the evaporation front towards the internal surface of the plate. At boiling there is much more vapour formed at the inner surface than forced through the plate.

The type of intersection of the saturation line and curve 1 characterizes a stable system of two-phase transpiration cooling in which minor perturbation of the process parameters causes transition into a new developed state near the original one. In system 3 minor perturbation leads to transition into a new state that considerably differs from the original one. System 3 is unstable. Thus, for the process of two-phase transpiration cooling to be steady, the tangent slope to the pressure vs temperature curve in the supposed region with phase transition should algebraically be larger than the tangent slope to the saturation curve at the working point

$$\frac{\mathrm{d}P_e}{\mathrm{d}T_e} > \frac{\mathrm{d}P_s}{\mathrm{d}T_s}. \tag{26}$$

STATIC CHARACTERISTICS OF UNSTABLE HEAT-ENGINEERING SYSTEMS

From the solution of the steady-state system of differential equations (1)–(10) the temperature dependence of pressure is obtained which represents a static characteristic of an equilibrium model of two-phase transpiration cooling. Static characteristics are tried means of studying a periodic stability of dynamic systems. They may also be used to estimate the permissible slow variations at which the process stability survives. Nowadays two types of aperiodic instability of heat-engineering equipment are known: burnout and sudden great-amplitude change in mass flow rate of working fluid in heated channels. Ambiguity is peculiar to static characteristics of both processes. Pool boiling may be described by a boiling curve which is a plot of the specific heat released from a heating surface to liquid vs surface and saturation temperature difference. The samples of boiling curves are presented in Fig. 8 [31, 32]. The hydrodynamic characteristic of the heated channel, its resistance vs working fluid flow rate is presented in Fig. 9 [33, 34]. There is the descending section *CE* in both curves. For liquid boiling it plots an increase in heat flux released from the surface with decreasing temperature difference in the transient region between film and nuclear boiling. On hydrodynamic curve 1 section *CE* corresponds to a rise in channel resistance due to increase of the vapour content of a two-phase flow.

A stability of wall cooling by boiling is shown by intersection of the boiling curve and the curve of specific heat flux supplied from external sources against the wall temperature. The stability condition is formulated in an analytical form [35, 36] as

$$\frac{\mathrm{d}q_{\mathrm{ext}}}{\mathrm{d}T_w} < \frac{\mathrm{d}q_{\mathrm{int}}}{\mathrm{d}T_w}. \tag{27}$$

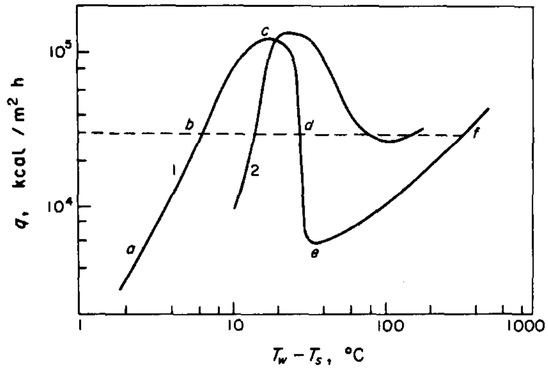

FIG. 8. Characteristic curves of pool boiling: 1, nitrogen [31]; Freon-113 [32].

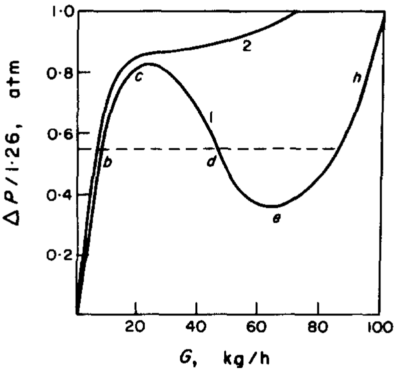

FIG. 9. Hydrodynamic characteristics of uniformly heated channel [33]; 1, non-single-values; 2, single-values.

In practice the processes (electric heating, radiation heating, heat release in fuel cells) are widely spread in which the heat flux supplied to the surface is quite or almost independent of its temperature $\mathrm{d}q_{\mathrm{ext}}/\mathrm{d}T_w = 0$. In cases involving such processes it is impossible to realize transient boiling for which $\mathrm{d}q_{\mathrm{int}}/\mathrm{d}T_w < 0$ because of a break in the stability conditions. The critical heat flux (point *C*) lays limits to stable and reliable operation. In case of transient boiling a steady state is achieved when it is somehow possible (for instance by vapour condensation) to provide a heat supply towards the plate with the heat transfer coefficient whose value is greater than that of the boiling curve slope.

However, because of limited heat-transfer coefficient and essential destabilizing effect due to the thermal wall resistance steady states are achieved only at the beginning and at the end of the transition region where the slope of the boiling curve is small [37, 38]. Therefore, to obtain the data for the whole transient region, either convective cooling of a heat releasing element in addition to cooling of its main heat-releasing surface by boiling at steady-state conditions [32, 39, 40] or unsteady-state cooling of a preliminarily heated body by boiling is performed [31, 32].

A liquid flow in a heated channel is stable if the slope of its hydrodynamic curve at the working point is algebraically greater than that of the characteristic of the pumping installation (the curve of flow rate *G* vs channel pressure ΔP_{ext} caused by external source [41])

$$\frac{\mathrm{d}\Delta P_{\mathrm{ext}}}{\mathrm{d}G} < \frac{\mathrm{d}\Delta P_{\mathrm{int}}}{\mathrm{d}G}. \tag{28}$$

If a constant pressure drop $\mathrm{d}\Delta P_{\mathrm{ext}}/\mathrm{d}G = 0$ is maintained within the heated channel, then the descending section of the channel characteristic presents the region of unstable operation. Aperiodic instability under these conditions may be avoided by eliminating ambiguity of the hydrodynamic characteristic by connecting the additional resistance in the form of a throttling washer to the entrance of the duct where one-phase liquid flows. Pressure drop on the washer and the duct resistance make the process stable, and the hydrodynamic characteristic acquires the form of curve 2 in Fig. 9.

It should be noted that the descending section of the hydrodynamic curve is peculiar not only to the system with phase conversions in the working fluid. It turns out that the hydrodynamic characteristic of a gas-cooled porous plate subject to surface heating [3] has the same form as curve 1 in Fig. 9. Here the ambiguity of the characteristic is due to temperature variation of the dynamic and kinematic viscosity of the gas coolant and results in three possible injectant mass flow rates corresponding to some definite pressure drop on the plate at considerably different mean wall temperature. The branch *EH* represents the region of stable and reliable operation where effective control of a coolant flow is possible by changing pressure drop on the plate and the flow rate is sufficient to keep the material temperature within permissible range. The reduction of the pressure drop below the value corresponding to the point E results in a sharp decrease of the coolant flow rate that involves the wall burnout. Unlike the system with phase conversions of a working fluid, no artificial measures can add inambiguity to the hydrodynamic characteristic of the porous gas-cooled plate. The ambiguity of the hydrodynamic characteristic of the system with a gaseous coolant was firstly found when investigation heat-releasing helium-cooled elements [42].

It follows from the above that static characteristics make it possible to determine the value of permissible slow variations of the parameters (density of the supplied heat flux for boiling, pressure drop on the plate for gaseous transpiration cooling) at which stable and reliable operation of the system survives.

As for two-phase transpiration cooling it makes no difficulty to find only permissible fluctuations of the initial coolant temperature using the earlier plotted static characteristic (23) which is the most natural result of solution of the system of equations describing the process. Because of the specific of the system operation, of the same importance is determination of the value of permissible fluctuations of such parameters as density of external heat flux and pressure on the plate.

HYDRODYNAMIC CHARACTERISTIC OF TWO-PHASE TRANSPIRATION COOLING

The title characteristic establishes a relationship between the pressure drop on the porous plate and the coolant flow rate for different values of the coordinate of the surface of equilibrium phase conversions. It is the condition of equilibrium evaporation which makes the basis for derivation of the required relationship.

Pressure within the interface region is found through resistance of the vapour region

$$P_e - P_1 = \delta\left[\alpha G v''(1-l) + \beta\frac{G^2}{\rho''}(1-l)\right]. \tag{29}$$

Instead of temperature T_s, calculated from (21), within the region of equilibrium phase transition it is of advantage to use the enthalpy i''_e of generating dry saturated vapour

$$i''_e - c_m T_\infty = q\frac{\exp\left[\dfrac{G\delta c''}{\lambda_2}(l-1)\right]}{G}. \tag{30}$$

Use of the vapour enthalpy and mean heat capacity of liquid in the temperature range of 0°C—T_∞ simplifies the expression and makes it more exact at the same time. The expression obtained is rewritten as

$$i''_e - i''_{e=1} = q\frac{\exp\left[\dfrac{G\delta c''}{\lambda_2}(l-1)\right]}{G} - (i''_{e=1} - c_m T_\infty) \tag{31}$$

in order to take the saturation parameters for evaporation at the external wall surface where the ambient pressure P_1 is known as the reference point for the enthalpy of the saturated vapour and pressure within the phase conversion region. Since both of these states are the states of equilibrium saturation, there exists a single-valued relationship between the difference of their pressures and enthalpy of dry vapour depending on the kind of a coolant

$$i''_e - i''_{e=1} = \psi(P_e - P_1). \tag{32}$$

The linear approximation of this relationship at the point where the pressure is equal to the ambient pressure

$$i''_e - i''_{e=1} = \left.\frac{\mathrm{d}i''}{\mathrm{d}P}\right|_{P=P_1}(P_e - P_1) \tag{33}$$

allows expressions (29) and (31) to be coupled into one analytical transcendental equation for determining the coolant flow rate G at equilibrium phase conversions in the region with the coordinate l

$$\frac{q\exp\left[\dfrac{G\delta c''}{\lambda_2}(l-1)\right]}{G} - (i''_{e=1} - c_m T_\infty) = \left.\frac{\mathrm{d}i''}{\mathrm{d}P}\right|_{P_1}\delta\left[\alpha G v''(1-l) + \beta\frac{G^2}{\beta''}(1-l)\right]. \tag{34}$$

The required pressure drop on the plate is calculated using the known flow rate from expression (19). The hydrodynamic characteristic of the two-phase transpiration cooling system is found from a successive solution of equations (34) and (19) for all values of the parameter l with its subsequent elimination. Physical properties of the porous wall and of both coolant phases as well as an external heat flux and initial liquid temperature are included as constant parameters of the process.

The solution of implicit equation (32) or even of its simplified analytical variant (34) involves tedious calculations. There is, however, one particular case being very promising for practical application when the length of calculations may be reduced. In accordance with the data of [43] the saturated steam enthalpy within 4·5 per cent remains constant i'' = const in a wide range of pressure from 1 to 120 bars. Characteristic equation (34) for the flow rate in the water-cooled system is simplified

$$q\frac{\exp\left[\dfrac{G\delta c''}{\lambda_2}(l-1)\right]}{G} = (i'' - c_m T_\infty). \tag{35}$$

The results of solution of equations (35) and (19) are presented in Fig. 10. Herein the constant parameters of the systems are left unchanged to be compared with the static characteristics in Fig. 6. The hydrodynamic characteristics are plotted in normalized coordinates. The pressure drop is referred to the value of $(P_0 - P_1)^a = 1{\cdot}25$ bar that provides equilibrium phase transition within the region with the coordinate $l = 0{\cdot}95$. The appropriate Reynolds number of a coolant flow is $Re = 0{\cdot}1$. Coolant flow rates are compared to $G_1^a = 4{\cdot}34\,\mathrm{kg/m^2.s}$ of the liquid flow rate subjected to the pressure drop $(P_0 - P_1)^a = 1{\cdot}25$ bar under viscous flow conditions ($Re = 0{\cdot}0$).

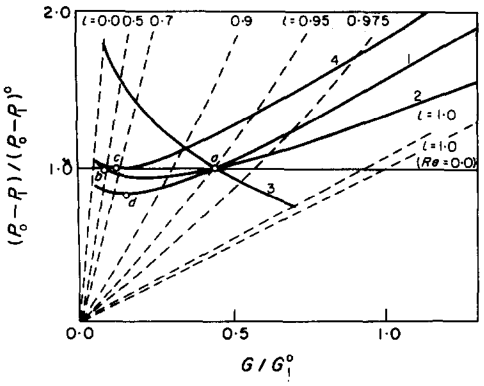

FIG. 10. Hydrodynamic characteristic of two-phase transpiration cooling system: 1, $q = 3{\cdot}5 \times 10^7\,\text{W/m}^2$, $\lambda_2 = 0{\cdot}65\,\text{W/m.deg}$; 2, $q = 1{\cdot}8 \times 10^7\,\text{W/m}^2$, $\lambda_2 = 1{\cdot}0\,\text{W/m.deg}$; 3, $q = 8{\cdot}3 \times 10^6\,\text{W/m}^2$, $\lambda_2 = 2{\cdot}6\,\text{W/m.deg}$; 4, $q = 8{\cdot}3 \times 10^7\,\text{W/m}^2$, $\lambda_2 = 0{\cdot}65\,\text{W/m.deg}$.

The inclined dash lines l = const relate the coolant flow rate and pressure drop on the plate for the fixed position of the phase conversion region. In particular, the line $l = 1{\cdot}0$ is for the plate resistance to a one-phase liquid flow and the line $l = 0{\cdot}0$, that to a dry vapour flow. Smooth deviation of dashed lines from the original straight line exhibits growth in the inertial resistance when the flow changes from viscous to transient (Fig. 10).

Hydrodynamic characteristics of all three systems intersect at the working point a because of the proper choice of the parameters λ_2, q. The working point of systems 1 and 2 is, however, within an ascending section, while that of system 3 lies within a descending section. The pressure drop on the plate is assumed to be constant $(P_0 - P_1) = 1{\cdot}25$ bar. Therefore, according to stability condition (28) systems 1 and 2 are stable at the working point a whereas system 3 is unstable. System 2 has one more working point b corresponding to unsteady-state operation.

It should be added that system 3 is unstable also in the case when the coolant evaporates on the external surface $l = 1{\cdot}0$. Therefore the calculation of the coolant flow rate from the heat balance equation does not appear to be quite correct for solution of thermal component of two-phase transpiration cooling without being examined for stability [11, 44–46].

Conclusions on the stability of the two-phase transpiration cooling made from the study of its hydrodynamic characteristic fully coincide with the results of an analysis for intersection of the saturation and pressure vs temperature curves in the supposed region with phase conversions. In addition, the value of permissible slow fluctuations of pressure drop on a plate may be found using the hydrodynamic characteristic of the stable system. Thus, in system 1 reduction of delivery pressure involves smooth recession of the evaporation front up to the stability boundary (point d). Afterwards, the evaporation region quickly propagates towards the inner surface of the plate causing a considerable decrease in the coolant flow rate. The point d establishes the minimum pressure drop for system 1.

Strictly speaking, in case of the assumed model with constant properties of both coolant phases chosen in a saturated state at prescribed ambient pressure the above method allows accurate measurement of pressure drop fluctuations only by fluctuations of the delivery pressure. A change in the ambient pressure involves variation of physical properties of coolant and, therefore, of the shape of the hydrodynamic characteristic. However an error due to permissible fluctuations of the controlling state may be supposed to be not larger than that implied by the model itself. Therefore permissible fluctuations of the ambient pressure may be found within the same accuracy using the characteristic suggested.

THERMAL CHARACTERISTIC OF TWO-PHASE TRANSPIRATION COOLING SYSTEM

Permissible fluctuations of external heat flux in the system of two-phase transpiration cooling may be determined by a thermal static characteristic, that is external heat flux density vs coordinate of equilibrium phase conversion surface. By transforming expression (30) we arrive at the main relation

$$q = (i_e'' - c_m T_\infty) G \exp\left[\frac{G\delta c''}{\lambda_2}(1-l)\right]. \tag{36}$$

The coolant flow rate G and enthalpy of saturation vapour i_e'' depend on the coordinate of phase transition surface and at constant pressure drop on a plate are calculated using expressions (18), (32), (29), (18), respectively.

Linear approximation of the dependence of saturated vapour enthalpy on pressure provides an analytical form of the desired static characteristic

$$q = G\exp\left[\frac{G\delta c''}{\lambda_2}(1-l)\right]\left\{(i_{e=1}'' - C_m T_\infty) + \frac{\mathrm{d}i''}{\mathrm{d}P}\bigg|_{P_1}\delta\left[\alpha G v''(1-l) + \beta\frac{G^2}{\rho''}(1-l)\right]\right\}. \tag{37}$$

For water taken as a coolant, the second summand in braces vanishes due to the reason mentioned above.

Figure 11 presents thermal characteristics 1–3 of those systems whose hydrodynamic characteristics 1–3 are plotted in Fig. 10. For all of the systems the wall pressure drop is constant $P_0 - P_1 = 1{\cdot}25$ bar. Heat flux in each system is based on the value q^a at which the phase transition region has the coordinate $l = 0{\cdot}95$. The respective normalizing values are given below the figure.

The working point a lies within the descending section of curves 1 and 2 and ascending section of curve 3. System 2 has the second working point b. For thermal characteristics the descending section is the stability region that follows from the physical nature of the process. Thus, a small increase in heat flux in stable system 1 against the developed state a causes recession of the interface surface which returns

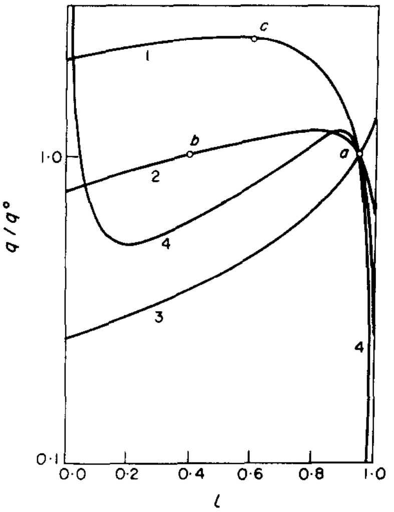

Fig. 11. Thermal characteristics (external heat flux density vs position of equilibrium phase conversion surface) of two-phase transpiration cooling: 1, $q = 3{\cdot}5 \times 10^7$ W/m^2, $\lambda_2 = 0{\cdot}65$ W/m.deg; 2, $q = 1{\cdot}8 \times 10^7$ W/m^2, $\lambda_2 = 1{\cdot}0$ W/m.deg; 3, $q = 8{\cdot}3 \times 10^6$ W/m^2, $\lambda_2 = 2{\cdot}6$ W/m.deg; 4, data of [44].

into the original state at subsequent relief of perturbation. In unstable system 3 the primary increase of outer heat flux causes continuous advance of the evaporation region towards the internal plate surface since in all of the intermediate states a smaller heat supply to external surface is required for equilibrium phase conversions to be achieved. With the use of the thermal characteristic the stability condition of two-phase transpiration cooling acquires the following form

$$\frac{dq}{dl} < 0. \tag{38}$$

It should be noted that the condition is valid for a system with any coolant, the plotting of the characteristic being simplified for water. A similar condition for this particular case is also derived from the solution of a linearized transient system of equations with a more simple statement of the whole problem [44]. In Fig. 11 curve 4 plots one of the results of this work. One more stability region for small l on curve 4 is accounted for by the unreal boundary condition of constant temperature at the inner surface of a permeable wall.

In a stable system the ultimate value of an external heat flux may readily be found using the thermal characteristic. A smooth growth of heat flux in system 1 causes gradual recession of the evaporation region up to the position with coordinate $l = 0{\cdot}6$ (point C) whereupon even a small perturbation leads to boiling on the inner surface. The hydrodynamic characteristic of system 1 with limited heat flux shows that the system is at the boundary of stability; curve 4 in Fig. 10 touches the line of constant pressure drop at the point C.

The study of aperiodic stability of the two-phase transpiration cooling system with the use of different static characteristics leads to similar results. Each of these characteristics allows permissible perturbation of one of the controlling parameters in a stable system to be easily detected. A simultaneous application of these characteristics makes it possible to find permissible fluctuations of all of the controlling parameters and to reveal in detail the peculiarities of the process.

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CARACTERISTIQUES STATIQUES DES SYSTEMES DE REFROIDISSEMENT PAR TRANSPIRATION, EN EQUILIBRE BIPHASIQUE

Résumé—On formule le problème du refroidissement biphasique permanent d'une plaque homogène poreuse. Il est montré qu'une relation approchée existe entre les composantes thermiques et hydrodynamiques du phénomène décrit par un système fermé non-linéaire d'équations différentielles. Les lois du mouvement de filtration d'un réfrigérant, avec ses changements de phase dans la paroi poreuse, sont analysées.

Les caractéristiques statiques du phénomène sont données sur la base des résultats obtenus à partir d'une solution du système d'équations qui décrit le refroidissement par transpiration biphasique. Il s'agit de la courbe de pression en fonction de la température dans la région suggérée du changement de phase, et des caractéristiques hydrodynamiques et thermiques. L'étude de la stabilité apériodique du phénomène à l'aide de ces caractéristiques fournit des résultats similaires, alors que l'utilisation d'un système de caractéristiques permet de trouver des perturbations admissibles de tous les paramètres et révèle des particularités du refroidissement par transpiration biphasique en régime permanent.

GLEICHGEWICHTSKENNLINIEN FÜR SYSTEME MIT SCHWITZWASSERKÜHLUNG

Zusammenfassung—Die Problemstellung bezieht sich auf die stationäre Zweiphasenkühlung einer homogenen, porösen Platte. Ein zufriedenstellender Ansatz zeigt die Zusammenhänge zwischen den thermodynamischen und hydrodynamischen Vorgängen des Prozesses, der durch ein gekoppeltes System nichtlinearer Differentialgleichungen beschrieben wird. Die Gesetze der Diffusionsgeschwindigkeit des Kühlmediums bei Phasenänderung in einer porösen Wand werden untersucht.

Mit den Ergebnissen, die sich aus der Lösung des Gleichungssystems für die Schwitzwasserkühlung ergeben, werden Kennlinien des Prozesses für den stationären Fall ermittelt. Diese Darstellungen sind als Kurvenläufe des Druckes mit der Temperatur innerhalb des betrachteten Bereiches der Phasenänderung aufgetragen. Die Betrachtung aperiodischer Gleichgewichtszustände des Prozesses mit Hilfe dieser Kennlinien liefert ähnliche Ergebnisse, wobei die gleichzeitige Anwendung mehrerer Kennlinien es erlaubt, mögliche Veränderungen aller kennzeichnenden Parameter herauszufinden und Eigenschaften der stationären Schwitzwasserkühlung aufzuzeigen.

СТАТИСТИЧЕСКИЕ ХАРАКТЕРИСТИКИ РАВНОВЕСНОЙ СИСТЕМЫ ДВУХФАЗНОГО ПОРИСТОГО ОХЛАЖДЕНИЯ

Аннотация — Сформулирована задача о стационарном двухфазном охлаждении однородной пористой пластины. Показана неразрывность тепловой и гидродинамической составляющих процесса, описываемого нелинейной замкнутой системой дифференциальных уравнений. Проанализированы закономерности фильтрационного движения охладителя при фазовом превращении его внутри пористой стенки.

На основе результатов решения системы уравнений, описывающей двухфазное пористое охлаждение, построены статические характеристики процесса: зависимость давления от температуры в предполагаемой области фазового превращения; гидродинамическая, тепловая. Исследование апериодической устойчивости процесса с помощью этих характеристик приводит к одинаковым результатам, а применение совокупности характеристик дает возможность обнаружить допустимые возмущения всех определяющих параметров и выявить особенности устойчивого двухфазного пористого охлаждения.